NUMERICAL INTEGRATION

Synopsis:

1. **Trapezoidal Rule :** Let y = f(x) be a continuous function defined on [a, b]. Divide the interval on x-axis into n equal parts each of length $h = \frac{b-a}{n}$. Let a = $x_0, x_1, x_2, \ldots, x_n$ =b be the abscissae of the successive points of division and $y_r = f(x_r)$ for r = 0, 1, 2, …, n. Then $\int_a^b f(x)\,dx =$

$$= \frac{h}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)]$$

$$= \frac{h}{2}[f(x_0) + f(x_n) + 2\{f(x_1) + f(x_2) + \ldots + f(x_{n-1})\}]$$

$$= \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})]$$

2. **Simpson's Rule :** Let y = f(x) be a continuous function defined on [a, b]. Divide the interval on x-axis into n = 2m (even integer) equal parts each of length $h = \frac{b-a}{n} = \frac{b-a}{2m}$. Let a = $x_0, x_1, x_2, \ldots, x_n = x_{2m}$ = b be the abscissae of the successive points of division and $y_r = f(x_r)$ for r = 0, 1, 2…n. Then

$$\int_a^b f(x)\,dx = \frac{h}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{2m-1}) + f(x_{2m})]$$

$$= \frac{h}{3}[\{f(x_0) + f(x_{2m})\} + 4\{f(x_1) + f(x_3) + \ldots + f(x_{2m-1})\}$$

$$+ 2\{f(x_2) + f(x_4) + \ldots + f(x_{2m-2})\}]$$

$$= \frac{h}{3}[(y_0 + y_{2m}) + 4(y_1 + y_3 + \ldots + y_{2m-1})] +$$

$$2(y_2 + y_4 + \ldots + y_{2m-2})]$$